TABLE I. Relations between channel amplitudes.

Process	Relations	Number of independent amplitudes
$PB\to PB$, $VB\to VB$	$A_{as}=A_{sa}$	7
$\bar{B}B\leftrightarrow PP$, $\bar{B}B\leftrightarrow VV$	$B_{\overline{10}}=B_{10}$	7
$BB\to BB$	$A_{as}=A_{sa}=0$	6
$\bar{B}B\to\bar{B}B$	$C_{\overline{10}}=C_{10}$, $C_{as}=C_{sa}$	6
$PP\to PP$, $VV\to VV$	$A_{\overline{10}}=A_{10}$, $A_{as}=A_{sa}=0$	5
$PV\to PV$, $PP\leftrightarrow VV$	(R conjunction invariance)	

and crossing symmetry lead to R invariance. This result holds because the octets P and V contain their charge conjugates. The results are summarized in Table I.

One can use the above results for the channels with nucleon number zero (except for $\bar{B}+B\to P+V$) in order to obtain some spin selection rules in a simple way. Let s be the magnitude of the total intrinsic spin of the system. Then for any state $|ab)$ one has[5]

$$CP|ab)=(-1)^{s}RE|ab)\quad \text{for } VV \text{ and } PP(s=0) \text{ systems,} \tag{6}$$
$$CP|ab)=(-1)^{s+1}RE|ab) \text{ for the } \bar{B}B \text{ system,}$$

where $E|ab)=|a'b')$, in which $a'(b')$ is the particle corresponding to $b(a)$ in the octet to which $a(b)$ belongs. For example, $E|p_\alpha\bar{\Sigma}_\beta{}^+)=|\Sigma_\alpha{}^-\bar{\Xi}_\beta{}^-)$, where the indices α and β indicate the spin state of each particle. (In particular, for the PP system, the E operation is equivalent to the particle exchange.)

Now CP invariance and Eq. (6) lead to

$$(ab|cd)=\pm(RE(ab)|RE(cd)). \tag{7}$$

The sign to be used depends on the initial and final values of s. In terms of channel amplitudes, the negative sign in Eq. (7) means that $A=0$ except for A_{10} and $A_{\overline{10}}$ for which $A_{10}=-A_{\overline{10}}$. However, as has been shown before, time-reversal invariance in the crossed channel of the processes considered here and the crossing symmetry lead to $A_{10}=A_{\overline{10}}$ so that the case of negative sign in Eq. (7) also implies that $A_{10}=A_{\overline{10}}=0$. Hence all the amplitudes are zero. Therefore, the sign in Eq. (7) should always be positive,[6] and one obtains the spin selection rules listed in Table II. Here one can see, for example, that a nucleon-antinucleon pair can decay into two pseudoscalar mesons only from spin-triplet states.

TABLE II. Spin selection rules.

Process	$\bar{B}B\to\bar{B}B$	$VV\to VV$	$VV\leftrightarrow PP$	$\bar{B}B\leftrightarrow PP$	$\bar{B}B\leftrightarrow VV$
Selection rule[a]	$s_i=s_f$	$s_i+s_f=$even	$s=0, 2$	$s=1$	$s_i+s_f=$odd

[a] The indices i and f denote the initial and the final states, respectively, and s means the magnitude of the total intrinsic spin.

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[5] This follows from the fact that: $C|ab)=R|ab)$ and $P|ab)=(-1)^{s}E|ab)$ for VV and $PP(s=0)$ systems, $C|ab)=(-1)^{s+L}\times RE|ab)$ and $P|ab)=-(-1)^{L}|ab)$ for the $\bar{B}B$ system, where L is the magnitude of the orbital angular momentum.

[6] This implies the invariance under the RE operation and leads to $A_{\overline{10}}=A_{10}$, which is consistent with the result obtained before.

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Three-Particle Unitarity Integral*

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Various forms of the production amplitude are proposed which are convenient for the three-particle unitarity integral. Both exact and approximate forms are discussed. It is found that in both cases one can separate the final-state configuration from the over-all kinematics by using discrete variables. Form factors with the three-particle intermediate state are also discussed.

I. INTRODUCTION

ONE of the outstanding difficulties in dispersion theory is the problem of unitarity integral involving more than two particles in the intermediate state.[1] Although many attempts have been made to amend this difficulty, the crude two-particle approximation seems to be the only method giving useful results.[2] In the case of three-particle intermediate state, various authors considered simple Feynman diagrams to study analytic properties of the absorptive part.[3] It

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[1] See, for instance, G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin, Inc., New York, 1961).

[2] L. F. Cook and B. W. Lee, Phys. Rev. **127**, 283 (1962).

[3] V. N. Gribov and I. T. Dyatlov, Zh. Eksperim. i Teor. Fiz. **42**, 196 (1962); **42**, 1268 (1962) [English transls.: Soviet Phys.—JETP **15**, 140 (1962); **15**, 879 (1962)]; Y. S. Kim, Phys. Rev. **132**, 927 (1963).

should be noted, however, that the Feynman amplitudes considered there do not in any way produce the observed three-body resonances. It should also be noted that the mathematical difficulties in such an analysis completely obscure possible physical insights. We feel thus that the perturbation theoretic approach does not give any useful result.

In this note we present various schemes for evaluating the three-particle absorptive part from experimentally measured quantities. We first re-examine the kinematical problems and propose a partial-wave expansion which is convenient for performing the unitarity integral and also for making contact with the experimental data. Then various approximate forms are discussed. The comparison with the two-particle approximation is made.

In Sec. II, we write the absorptive part of the two-body scattering amplitude due to the three-particle intermediate state. In Sec. III, a partial-wave expansion is proposed for the production amplitude. We write the unitarity integral as a sum and an integral over the directional Dalitz plots. In Sec. IV, an approximation scheme is proposed in which only resonance terms are retained for the production amplitude. The unitarity integral is written as a trace of the product of two matrices, one corresponding to the final-state structure, the other to the orientation of the initial state. In Secs. V and VI, another approximation scheme is discussed, which is convenient for the calculation of form factors. The structure constants are written in terms of the resonance parameters.

II. THREE-PARTICLE CONTRIBUTION IN TWO-BODY SCATTERING

In a dispersion theoretic discussion of the two-body scattering of spinless bosons,

$$p_1+p_2\to p_1'+p_2',$$

where p_1 and p_1' are the four-momenta of the incoming and outgoing particles, we are led to consider the absorptive part of the scattering amplitude. The unitarity condition enables us to write the absorptive part as a sum over all possible intermediate states. Then we make an approximation of retaining only a few lowest mass terms in the summation. The mathematical structures of the one- and two-particle terms are well known. Due to the difficulties mentioned in the preceding section, the structure of the three-particle term has not been thoroughly examined. In the following discussions we shall study the three-particle unitarity integral for various cases. In order to guarantee proper reality conditions at every stage of approximation we adopt the $\frac{1}{2}\{|\text{in}\rangle+|\text{out}\rangle\}$ convention for the summation over the intermediate states.

Now the contribution to the absorptive part from the three-particle state is

$$\begin{aligned}\operatorname{Im}A^{(3)}&(p_1+p_2\to p_1'+p_2')\\ &=\frac{1}{2}\left(\frac{1}{2\pi}\right)^5\operatorname{Re}\int d^3k_1d^3k_2d^3k_3\,A(p_1+p_2\to k_1+k_2+k_3)\\ &\quad\times A^*(p_1'+p_2'\to k_1+k_2+k_3)\delta(k_1^2+\mu^2)\delta(k_2^2+\mu^2)\\ &\quad\times\delta(k_3^2+\mu^2)\theta(k_{10})\theta(k_{20})\theta(k_{30})\\ &\qquad\times\delta(p_1+p_2-k_1-k_2-k_3)\,,\end{aligned}\quad(1)$$

where $A(p_1+p_2\to k_1+k_2+k_3)$ is the invariant amplitude for the production process

$$p_1+p_2\to k_1+k_2+k_3\,.$$

We are assuming that all particles have the same mass μ.

$$k_i^2+\mu^2=p_i^2+\mu^2=0\,.$$

It is often more convenient to discuss the partial-wave projection of the above absorptive part.

$$\operatorname{Im}A_l^{(3)}(W)\equiv\int_{-1}^{1}d(\cos\theta)\,\operatorname{Im}A^{(3)}(p_1+p_2\to p_1'+p_2')\,,\quad(2)$$

where W and θ are, respectively, the total energy and the scattering angle in the center-of-mass system.

In the following sections we shall discuss $\operatorname{Im}A_l^{(3)}(W)$ using various forms of the production amplitude.

III. EXPANSION IN SPHERICAL HARMONICS

For the production amplitude

$$A(p_1+p_2\to k_1+k_2+k_3)\,,\quad(3)$$

we use the variables

$$S_1=-(k_2+k_3)^2,\quad S_2=-(k_1+k_3)^2,\quad S_3=-(k_1+k_2)^2\quad(4)$$

and the redundant variable

$$\begin{aligned}W^2&=-(p_1+p_2)^2=-(p_1'+p_2')^2=-(k_1+k_2+k_3)^2\\ &=S_1+S_2+S_3-3\mu^2\end{aligned}$$

to describe the three-particle final state. The above amplitude depends, in addition, on two other variables which describe the orientation of the final-state configuration with respect to the direction of the incoming beam. In the center-of-mass system, we can write the amplitude as

$$A(p_1+p_2\to k_1+k_2+k_3)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}Y_{lm}(\alpha,\beta)A^{lm}(W^2,S_1,S_2,S_3)\,,\quad(5)$$

where $Y_{lm}(\alpha,\beta)$ are the spherical harmonics. α and β are the standard polar angles of the vector $\mathbf{k}_1$ with respect to the incoming beam. One can, of course, replace the former by any other convenient direction in the final-state configuration. The partial-wave amplitude $A^{lm}(W^2,S_1,S_2,S_3)$ is now a function only of the final-state variables.

Without loss of generality we can assume that the two-body outgoing beam is in the XZ plane and makes the scattering angle θ with the incoming direction. Then

$$A(p_1'+p_2'\to k_1+k_2+k_3)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}[R(\theta)Y_{lm}(\alpha,\beta)]A^{lm}(W^2,S_1,S_2,S_3)\,,\quad(6)$$

where $R(\theta)$ is the operator that rotates $Y_{lm}(\alpha,\beta)$ around the Y axis. In terms of the Wigner rotation matrix $d(\theta)$,[4] one can write

$$R(\theta)Y_{lm}(\alpha,\beta)=\sum_{m} d_{m'm}{}^{l}(\theta)Y_{lm'}(\alpha,\beta)\,.\quad(7)$$

We now substitute Eqs. (5) and (6) into Eq. (1) and perform the angular integrations to obtain

$$\mathrm{Im}A_l{}^{(3)}(W)=\frac{1}{16}\left(\frac{1}{2\pi}\right)^5\mathrm{Re}\sum_{m,\mu}F_{\mu m}{}^{l}\int_{\mu}^{(W^2-3\mu^2)/2W}dk_{10}\times\int_{k_{20}{}^{(-)}}^{k_{20}{}^{(+)}}dk_{20}A^{l\mu}(W^2,S_1,S_2,S_3)\times[A^{lm}(W^2,S_1,S_2,S_3)]^*\,,\quad(8)$$

where

$$F_{\mu m}{}^{l}=\int_{-1}^{1}d(\cos\theta)P_l(\cos\theta)d_{\mu m}{}^{l}(\theta)\,,$$

$$k_{20}{}^{(\pm)}=\tfrac{1}{2}(W-k_{10})\pm\frac{1}{2}\left[(k_{10}{}^2-\mu^2)\frac{W^2-3\mu^2-2Wk_{10}}{W^2+\mu^2-2Wk_{10}}\right]^{1/2}.$$

In terms of the integration variables, S_1, S_2, and S_3 take the form

$$S_1=W^2+\mu^2-2Wk_{10}\,,$$
$$S_2=W^2+\mu^2-2Wk_{20}\,,$$
$$S_3=2(k_{10}+k_{20})W+\mu^2-W^2\,.$$

According to the above expression, we first integrate over each directional Dalitz plot $A^{l\mu}(A^{lm})^*$ and then sum over the directional number m. In order to calculate $\mathrm{Im}A_l{}^{(3)}(W)$ one has to determine all $(2l+1)^2$ directional Dalitz plots. However, the experiment so far has not been oriented toward such a measurement. In the following section we shall discuss an approximate form of the production amplitude more convenient for the experimental determination of $\mathrm{Im}A_l{}^{(3)}(W)$.

IV. RESONANCE EXPANSION

The characteristic feature of production processes is the existence of competing final-state interactions.[5] We now make an approximation of retaining only terms giving rise to the observed resonances. Assuming for simplicity that there is one resonance for each pair of particles, we can write the production amplitude of Eq. (5) in the complex-propagator form

$$A(p_1+p_2\to k_1+k_2+k_3)\simeq\left[\frac{h_1(\alpha,\beta,W^2)}{S_1-m_1{}^2}+\frac{h_2(\alpha,\beta,W^2)}{S_2-m_2{}^2}+\frac{h_3(\alpha,\beta,W^2)}{S_3-m_3{}^2}\right].\quad(9)$$

The complex constant m_1 is chosen in such a way that the above expression would predict the observed resonance in the S_1 channel. m_2 and m_3 are chosen in a similar way. $h_i(\alpha,\beta,W^2)$ describes the dependence of the S_i component on the orientation with respect to the incoming beam. In addition, $h_i(\alpha,\beta,W^2)$ is to predict the resonance in the over-all three-body system. Similarly, we write

$$A(p_1'+p_2\to k_1+k_2+k_3)\simeq\left[\frac{R(\theta)h_1(\alpha,\beta,W^2)}{S_1-m_1{}^2}+\frac{R(\theta)h_2(\alpha,\beta,W^2)}{S_2-m_2{}^2}+\frac{R(\theta)h_3(\alpha,\beta,W^2)}{S_3-m_3{}^2}\right].\quad(10)$$

The operator $R(\theta)$ is defined in Eq. (7). $h_i(\alpha,\beta,W^2)$ is, in general, a linear combination of the spherical harmonics.

Substituting Eqs. (9) and (10) into Eq. (1), we obtain

$$\mathrm{Im}A_l{}^{(3)}(W)=\frac{1}{16}\left(\frac{1}{2\pi}\right)^5\mathrm{Re}\{\mathrm{Tr}[H^l(W)G(W)]\}\,,\quad(11)$$

where

$$H_{ij}{}^{l}(W)=\int_{-1}^{1}d(\cos\theta)P_l(\cos\theta)\int_{0}^{\pi}d(\cos\alpha)\times\int_{0}^{2\pi}d\beta h_i(\alpha,\beta)[R(\theta)h_j(\alpha,\beta)]^*\,,\quad(12)$$

$$G_{ij}(W)=\int_{\mu}^{(W^2-3\mu^2)/2W}dk_{10}\times\int_{k_{20}{}^{(-)}}^{k_{20}{}^{(+)}}\frac{dk_{20}}{(S_i-m_i{}^2)^*(S_j-m_j{}^2)}\,.\quad(13)$$

The matrix $G(W)$ depends only on the parameters of the final-state resonances. The matrix $H(W)$, on the other hand, describes the angular dependence of each resonance and the over-all three-particle interaction. Here again we have a convenient form in which the final-state terms are explicitly factored out.

Let us add a few remarks. By tracing back the calculation which led us to Eq. (11), one can easily see that the two-particle approximation corresponds to ignoring all off-diagonal elements of $G(W)$, $H(W)$ or both. We note, however, that the matrix $G(W)$ is Hermitian. Thus, we can always diagonalize it and reduce the original problem to an effective two-body unitarity integral.

So far we have confined ourselves only to the S-wave final-state interactions. If some resonances mentioned above are of the P- or higher-wave type, then the only

[4] See, for instance, M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

[5] Y. S. Kim, Phys. Rev. **125**, 1771 (1962); R. F. Peierls and J. Tarski, *ibid.* **129**, 981 (1963).

modification is that a polynomial in the integration variables is introduced in the numerator of the integrand in Eq. (13). In any case, the k_{20} integral can be performed by partial fraction.

In the preceding discussions, we have presented two possible forms of the production amplitude which are convenient for the experimental determination of the three-particle absorptive part. In the following section, we shall consider another approximate form in connection with form factor calculations.

V. FORM FACTORS

We take as an example the pionic form factor of nucleon. Its relativistic invariant form is

$$I(t)=(2p_0 2p_0')^{1/2}\langle p'|J(0)|p\rangle, \tag{14}$$

where p and p' are, respectively, the four-momenta of the incoming and outgoing nucleons. $J(0)$ is the pion current operator. Here again we ignore spins and isospins. The form factor is then a function only of the invariant variable $t=-(p-p')^2$.

The above pionic vertex function contains all essential features common to both electromagnetic and weak interaction form factors. It has been noted by many authors that the study of the quantity[6]

$$F(W)=(2p_0 2\bar{p}_0)^{-1/2}\langle 0|J(0)|p,\bar{p},\text{in}\rangle, \tag{15}$$

with $W^2=-(p+\bar{p})^2$, is completely equivalent to the original problem. The state $|p,\bar{p},\text{in}\rangle$ represents the nucleon-antinucleon "in" state with p and $\bar{p}$ as the four-momenta of the nucleon and antinucleon.

The function $F(W^2)$ satisfies the dispersion relation

$$F(W^2)=\frac{1}{\pi}\int_{9\mu^2}^{\infty}\frac{dW'^2A(W'^2)}{W'^2-W^2-i\epsilon}, \tag{16}$$

where

$$A(W^2)=\left(\frac{p_0}{2}\right)^{1/2}(2\pi)^4\sum_n\langle 0|J(0)|n\rangle \times\langle n|\bar{f}(0)|p\rangle\delta(\bar{p}+p-p_n).$$

$\bar{f}(0)$ is the antinucleon current operator. μ is the pionic mass. We have omitted a possible subtraction term which at worst can cause a polynomial increase in W^2. Here again the $\frac{1}{2}\{|\text{in}\rangle+|\text{out}\rangle\}$ convention is adopted for the summation over intermediate states.

We now make the approximation of retaining only the three-pion and nucleon-antinucleon intermediate states. The dispersion relation is then the integral equation

$$F(W^2)=\frac{1}{\pi}\int_{9\mu^2}^{\infty}\frac{dW'^2T(W')}{W'^2-W^2-i\epsilon}+\frac{1}{\pi}\int_{4m^2}^{\infty}dW'^2 \times\frac{\text{Re}\{F(W'^2)[e^{i\delta(W'^2)}\sin\delta(W'^2)]^*\}}{W'^2-W^2-i\epsilon}, \tag{17}$$

where $\delta(W^2)$ is the S-wave phase shift for the nucleon-antinucleon scattering. We use m for the nucleon mass. $T(W')$ is the three-particle contribution to the unitarity sum.

$$T(W^2)=\left(\frac{1}{2\pi}\right)^5\left(\frac{p_0}{2}\right)^{1/2} \times\text{Re}\int d^3k_1d^3k_2d^3k_3\langle 0|J(0)|k_1,k_2,k_3,\text{out}\rangle \times\langle k_1,k_2,k_3,\text{out}|\bar{f}(0)|p\rangle\delta(p+\bar{p}-k_1-k_2-k_3).$$

k_1, k_2, and k_3 are the four-momenta of the three intermediate pions. For the three-particle amplitude in the integrand, one can choose approximate forms in the way discussed before. However, let us consider here another possibility.

A crude form for the three-particle amplitude will be a product of all complex propagators corresponding to the observed resonances. This form of course has many serious drawbacks. For instance, the amplitude so constructed cannot describe the orientation of the initial beam with respect to the final-state configuration. We nevertheless use this form primarily because the present experimental uncertainty does not allow anything better.

As in the previous section we assume that there is one resonance for each pair of pions. Then

$$\langle k_1,k_2,k_3,\text{out}|\bar{f}(0)|p\rangle =(2k_{10}2k_{20}2k_{30}2p_0)^{-1/2}\frac{\Gamma_1}{(S_1-m_1^2)(S_2-m_2^2)(S_3-m_3^2)},$$

$$\langle 0|J(0)|k_1,k_2,k_3,\text{out}\rangle =(2k_{10}2k_{20}2k_{30})^{-1/2}\left[\frac{\Gamma_2}{(S_1-m_1^2)(S_2-m_2^2)(S_3-m_3^2)}\right]^*.$$

The complex constants m_1, m_2, and m_3 are resonance parameters. The constants Γ_1 and Γ_2 are assumed to be real.

Then after the angular integrations,

$$T(W^2)=\frac{\pi^2\Gamma_1\Gamma_2}{2}\left(\frac{1}{2\pi}\right)^5\int_{\mu}^{(W^2-3\mu^2)/2W}dk_{10} \times\int_{k_{20}^{(-)}}^{k_{20}^{(+)}}dk_{20}\left|\frac{1}{m_3^2+W^2-\mu^2-2Wk_{10}-2Wk_{20}}\right|^2 \times\left|\frac{1}{(m_1^2-\mu^2-W^2+2Wk_{10})(m_2^2-\mu^2-W^2+2Wk_{20})}\right|^2, \tag{18}$$

$$k_{20}^{(\pm)}=\tfrac{1}{2}(W-k_{10})\pm\frac{1}{2}\left[(k_{10}^2-\mu^2)\frac{W^2-3\mu^2-2Wk_{10}}{W^2+\mu^2-2Wk_{10}}\right]^{1/2}.$$

One can now perform the k_{20} integral by partial fraction. The k_{10} integral is to be done by a numerical method, but is within the capacity of high-speed computers.

[6] P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. **112**, 642 (1958).

VI. INTEGRAL EQUATION FOR FORM FACTORS

Let us regard the three-particle term in Eq. (17) as a known function. Then

$$F(x)=g(x)+\frac{1}{\pi}\int_{4m^2}^{\infty}dx'\frac{\mathrm{Re}[F(x')e^{-i\delta^*(x')}\sin\delta^*(x')]}{x'-x-i\epsilon}, \quad (19)$$

where $x=W^2$, and

$$g(x)=\frac{1}{\pi}\int_{9\mu^2}^{\infty}\frac{dx'T(x')}{x'-x-i\epsilon}.$$

The only difference from the conventional Omnes integral equation is that the inhomogeneous term has a cut extending from $9\mu^2$ to ∞. This overlaps with the homogeneous cut of the second term. The above equation can nevertheless be solved by an elementary method if the phase shift $\delta(x')$ is real throughout the region of integration. Let us assume that $\delta(x')$ is real and solve the integral equation.

We first introduce the auxiliary function

$$\Phi(x)=\frac{1}{2\pi i}\exp\left[-\frac{1}{\pi}\int_{4m^2}^{\infty}\frac{dx'\delta(x')}{x'-x-i\epsilon}\right]\times\int_{4m^2}^{\infty}dx'\frac{\mathrm{Re}[e^{-i\delta(x')}\sin\delta(x')F(x')]}{x'-x-i\epsilon}. \quad (20)$$

The function $\Phi(x)$ has a cut extending from $4m^2$ to ∞ with the discontinuity

$$\Phi(x+i\epsilon)-\Phi(x-i\epsilon)=\tan\delta(x)[\cos\delta(x)\,\mathrm{Re}g(x)+\sin\delta(x)\,\mathrm{Im}g(x)]\times\exp\left[-\frac{1}{\pi}P\int_{4m^2}^{\infty}\frac{dx'\delta(x')}{x'-x-i\epsilon}\right]. \quad (21)$$

Then the solution follows immediately.

$$F(x)=g(x)+\frac{1}{\pi}\exp\left[\frac{1}{\pi}\int_{4m^2}^{\infty}\frac{dx'\delta(x')}{x'-x-i\epsilon}\right]\times\int_{4m^2}^{\infty}dx'\frac{\tan\delta(x')[\cos\delta(x')\,\mathrm{Re}g(x')+\sin\delta(x')\,\mathrm{Img}(x')]}{x'-x-i\epsilon}\times\exp\left[-\frac{1}{\pi}P\int_{4m^2}^{\infty}\frac{dx''\delta(x'')}{x''-x'}\right]. \quad (22)$$

It was noted by many authors that the effect of the nucleon-pair state is extremely small for physically interesting values of χ.[7] We thus ignore the second term and retain only the inhomogeneous part $g(x)$. Then in the expansion

$$F(x)=F(0)\{1+a_1x+a_2x^2+\cdots\}$$

for small x,

$$a_n=\left(\int_{9\mu^2}^{\infty}\frac{dx}{x^{n+1}}T(x)\right)\Big/\left(\int_{9\mu^2}^{\infty}\frac{dx}{x}T(x)\right),\quad n=1,2,\cdots. \quad (23)$$

We have obtained a closed form for the structure constants a_n. The advantage of using the pure product representation is that the coefficients a_n do not depend on the constant $(\Gamma_1\times\Gamma_2)$. They are functions only of the resonance parameters.

VII. CONCLUDING REMARKS

We have considered various forms of the production amplitude which are convenient for the three-particle unitarity integral. First, a partial-wave expansion has been considered. It is found that the lth partial-wave term is the sum of $(2l+1)$ integrals over the directional Dalitz plot. Next, a resonance approximation has been discussed. Here, also, a factorization is achieved which separates out the final-state configuration from the over-all kinematics. Finally, a product representation of the complex propagator has been considered for form factor calculations. We have obtained an expression for the structure constant in terms of the resonance parameters.

The present work shows that one can simplify the three-body and possibly many-body kinematics by introducing discrete variables such as l, m in Sec. III and i, j in Sec. IV. The discrete variables in this paper are chosen in such a way that the dynamics of the final-state interaction is factored out.

Another remarkable point is that we can reduce the approximation scheme of Sec. IV to the two-body problem without ignoring the interference terms. It would be extremely interesting to see whether such a reduction can be achieved for the problems with more than three particles.

[7] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).